Method of determining filter coefficients from line spectral frequencies

The present invention relates to a method of determining filter coefficients from Line Spectral Frequencies comprising recomputing $P(z)$ and $Q(z)$ polynomials and comprising calculating the ω_i coefficients.

The coding of speech signals is used particularly in the field of mobile communications since the coded speech signal can be transmitted in a manner in which the redundancy commonly experienced in human speech is reduced. Linear Predictive Coding (LPC) is a known technique normally used in speech coding and in which the correlation of the speech signal is removed by means of a filter. The filter is best described by way of one of a different set of parameters, and one important set of which comprises LSFs.

An accurate representation of the filter is an important requirement since such information is transmitted with the speech signal for subsequent reconstruction of the speech signal at a signal-receiving unit.

The advantages of representing LPC filter coefficients in the form of LSFs have been well-documented since the inception of this concept in 1975. As is well known, the representation of an inverse LPC filter $A(z)$ in the form of LSFs is derived from the representation of $A(z)$ by its set of zeros in the z-plane. Insofar as the function $A(z)$ represents an all-zero filter, it can be fully and accurately described by way of reference to its corresponding set of zeros.

Computation of the LSFs commences with the decomposition of the polynomial $A_m(z)$ of order m into two inverse polynomial functions $P(z)$ and $Q(z)$. For confirmation, the polynomial $A_m(z)$ and the two inverse polynomials appear as follows:

$$A_m(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_m z^{-m}$$

and

$$P(z) = A_m(z) + z^{-(m+1)} A_m(z^{-1})$$

$$Q(z) = A_m(z) - z^{-(m+1)} A_m(z^{-1})$$

The polynomials $P(z)$ and $Q(z)$ each have $(m+1)$ zeros and exhibit various important characteristics. In particular:

all zeros of $P(z)$ and $Q(z)$ are found on the unit circle in the z-plane;

the zeros of $P(z)$ and $Q(z)$ are interlaced on the unit circle and the zeros do not overlap; and

the minimum phase property of $A_m(z)$ is easily preserved when the zeros of

5    $P(z)$ and $Q(z)$ are quantised.

Analysis of the above confirms that $z = -1$ and $z = +1$ is always zero with the functions $P(z)$ and $Q(z)$ and since these zeros do not contain any information relating to the LPC filter, they can simply be removed from $P(z)$ and $Q(z)$ by dividing by $(1 + z^{-1})$ and $(1 - z^{-1})$.

10   Such revised functions can be represented when m is even as follows:

$$P'(z) = \frac{P(z)}{(1 + z^{-1})} \text{ and } Q'(z) = \frac{Q(z)}{(1 - z^{-1})}$$

and when m is odd as:

$$P'(z) = P(z) \text{ and } Q'(z) = \frac{Q(z)}{(1 - z^{-1})(1 + z^{-1})}.$$

The advantageous properties of functions $P(z)$ and $Q(z)$ as noted above are

15   also valid for $P'(z)$ and $Q'(z)$. Since the coefficients of $P'(z)$ and $Q'(z)$ comprise real numbers, the zeros form complex conjugate pairs such that the search for zeros only has to be conducted on the upper half of the unit circle, i.e. where $0 < \omega < \pi$.

It generally proves inconvenient to compute complex zeros, particularly by way of computerised numerical analysis methods, and so the functions $P'(z)$ and $Q'(z)$ are

20   transformed to functions $P''(z)$ and $Q''(z)$ with real zeros. Also, the functions $P'(z)$ and $Q'(z)$ always have an even order and, since they are symmetrical, the functions can be re-written with real zeros to the following manner:

$$P''(\omega) = 2\sum_{i=0}^{m_p} p_i'' \cos((m_p - i)\omega)$$

$$Q''(\omega) = 2\sum_{i=0}^{m_q} q_i'' \cos((m_q - i)\omega)$$

25   where $p_0'' = 1$, $p_{1,2...m_p-1}'' = p_{1,2...m_p-1}'$, $p_{m_p}'' = \frac{1}{2}p_{m_p}'$, $q_0'' = 1$, $q_{1,2...m_q-1}'' = q_{1,2...m_q-1}'$,

$q_{m_q}'' = \frac{1}{2}q_{m_q}'$, and where m_p is equal to the number of zeros of $P'(z)$ on the upper half of

the unit circle and where m_q is equal to the number of zeros of $Q'(z)$ on the upper half of the unit circle.

When seeking the zeros of these functions, advantage can be taken from the form of the representations for $P''(z)$ and $Q''(z)$ due to the fact that the number of zeros to

5   be located is already known.

Importantly, and of particular relevance to the present invention, once the LSFs have been identified and employed as required, the recomputation of the LPC filter coefficients from the LSFs is required. While this stage represents a much less computationally intensive calculation than the computation of the LSFs from the filter

10   coefficients as discussed above, problems and disadvantageous limitations are experienced. In particular, the values of intermediate coefficients can be disadvantageously high and this can lead to numerical problems even when employing floating point representations.

Recomputing LPC filter coefficients a_i from LSFs is much less computationally intensive than computing the LSFs from the filter coefficients. Each LSF

15   ω_i, $i = 0,1,...,m-1$ contributes to a quadratic factor of the form, $1 - 2\cos(\omega_i)z^{-1} + z^{-2}$. The polynomials $P'(z)$ and $Q'(z)$ are formed by multiplying these factors using the LSFs that come from the corresponding polynomial:

$$P'(z) = \prod_{i=0}^{m_p-1}(1 - 2\cos(\omega_{2i})z^{-1} + z^{-2})$$

$$Q'(z) = \prod_{i=1}^{m_q-1}(1 - 2\cos(\omega_{2i+1})z^{-1} + z^{-2})$$

20   The polynomials $P(z)$ and $Q(z)$ are computed by multiplying $P'(z)$ and $Q'(z)$ with the extra zeros at $z = -1$ and $z = +1$. Finally, the filter coefficients are computed by using the following equation:

$$A_m(z) = \frac{P(z) + Q(z)}{2}$$

which defines the relationship between the polynomial $A_m(z)$ and the two inverse

25   polynomials discussed earlier.

Thus, when recomputing the $P(z)$ and $Q(z)$ polynomials one can use the above equations for $P'(z)$ and $Q'(z)$ and add the extra zeros. Thus, for m is even:

$$P(z) = (1 + z^{-1})\prod_{i=0}^{m_p-1}(1 - 2\cos(\omega_{2i})z^{-1} + z^{-2})$$

$$Q(z) = (1 - z^{-1}) \prod_{i=0}^{m_q-1} (1 - 2\cos(\omega_{2i+1})z^{-1} + z^{-2})$$

and for m is odd:

$$P(z) = \prod_{i=0}^{m_p-1} (1 - 2\cos(\omega_{2i})z^{-1} + z^{-2})$$

$$Q(z) = (1 - z^{-1})(1 + z^{-1}) \prod_{i=0}^{m_q-1} (1 - 2\cos(\omega_{2i+1})z^{-1} + z^{-2})$$

5   Since ω_i coefficients are ordered in increasing frequency the contributions of the first

$\cos(\omega_i)$ coefficients are positive and the last coefficients are negative. This introduces an

undesirable growth of the intermediate coefficient values while conducting the polynomial

multiplications $(1 - 2\cos(\omega_{2i+1})z^{-1} + z^{-2})$. With an increasing order of m, such problems are

amplified. To illustrate this, an example polynomial $Q(z) = 1 - z^{-2N}$ has been taken; it being

10   noted that that $m = 2N$. Such a polynomial provides for a system with *2N* equidistant zeros

on the unit circle. Since this is only a very simple example, it should of course be appreciated

that in reality the growth of the intermediate coefficients can be much larger. Therefore a

different strategy must be used. It has been found that for high order polynomials *m*=60 (or

N=30) a double precision floating point representation will not be sufficient. A different

15   method might involve searching for the best possible combination of ω_i and which exhibits

the least amount of intermediate coefficient growth. However, due to the large number of

possible combinations, this is most unlikely to be feasible and it also means that the optimal

combination will never be found.

The invention seeks to provide for a method of determining filter coefficients

20   having advantages over known such methods.

In accordance with the present invention there is provided a method of

determining filter coefficients from Line Spectral Frequencies as noted above and

characterised by the steps of addressing the polynomials in a series and reducing the number

of polynomials in ω_i in the said series by combining the polynomials in ω_i two by two in a

25   manner so as to arrive at two polynomials in ω_i and determining the product of the said two

polynomials.

The invention serves to combine ω_i in such a way that hardly any signal

growth occurs and proves particularly advantageous since the use of an increasing index *i*

would not seem to offer a good solution.

Using the method of the present invention with the example polynomial noted above, i.e. $Q(z) = 1 - z^{-2N}$, the intermediate coefficients are never larger than 2. In practice only a limited amount of intermediate coefficient growth occurs. Advantageously, the invention need not comprise a particularly complex method. In general it requires only a

5   different indexing and can advantageously deliver almost optimal results. For $P(z)$ the same procedure could be used only if m is even then $P(z)$ has a root at $z = -1$. If m is odd $P(z)$ does not have any additional roots, so no additional roots need to be added.

Finally the relationship noted above is employed to compute the coefficients of $A(z)$ from $P(z)$ and $Q(z)$.

The invention is described further hereinafter, by way of example only, with reference to the accompanying drawing which is a graphical representation of the intermediate coefficient growth experienced in the prior art and in an example polynomial

15      $Q(z) = 1 - z^{-2N}$.

It is assumed that the original polynomial is reconstructed by combining the zeros with increasing ω_i. The maximum value of the largest coefficient during the

20   recomputation procedure is plotted in the accompanying drawing. Note that the Y axis is logarithmic. For large order N the intermediate values of some of the coefficients become very high.

However, such problems are not encountered in a method according to the present invention.

As an example, and for $Q(z)$ with m is even, the following ordering of the polynomials is used:

$$v_0[0] = 1 - z^{-1}$$

$$v_0[1] = 1 - 2\cos\omega_1 z^{-1} + z^{-2}$$

$$v_0[2] = 1 - 2\cos\omega_3 z^{-1} + z^{-2}$$

30      $$v_0[m_q] = 1 - 2\cos\omega_{2 \cdot m_q - 1} z^{-1} + z^{-2}$$

If m is odd then the terms for $Q(z)$ are:

$$v_0[0] = 1 - z^{-1}$$

$$v_0[1] = 1 - 2\cos\omega_1 z^{-1} + z^{-2}$$

$$v_0[m_q] = 1 - 2\cos\omega_{2 \cdot m_q - 1} z^{-1} + z^{-2}$$

$$v_0[m_q + 1] = 1 + z^{-1}$$

5   The next step is to combine the polynomials $v_0[i]$. The strategy will be demonstrated with an example with $m=12$ and $m_q=6$. The original seven polynomials are: $v_0[0]$, $v_0[1]$, $v_0[2]$, $v_0[3]$, $v_0[4]$, $v_0[5]$, and $v_0[6]$.

In the first step the polynomials are combined two by two. Polynomial i is combined with polynomial $[m_q-i]$, this gives four intermediate polynomials $v_i[i]$:

10
$$v_1[0] = v_0[0] \cdot v_0[6]$$

$$v_1[1] = v_0[1] \cdot v_0[5]$$

$$v_1[2] = v_0[2] \cdot v_0[4]$$

$$v_1[3] = v_0[3]$$

These four polynomials are combined in the same way, leading to two new

15   polynomials $v_2[i]$:

$$v_2[0] = v_1[0] \cdot v_1[3]$$

$$v_2[1] = v_1[1] \cdot v_1[2]$$

The product $v_2[0].v_2[1]$ gives the final result:

$$v_3[0] = v_2[0] \cdot v_2[1]$$

20   The procedure can formally be described by the following pseudo program:

if m is even

begin

$m_q=m/2$

$m_c=m_q+1$

25   end

else

begin

$m_q=(m-1)/2$

$m_c=m_q+2$

30   end

$n_p=m_c$

```
i=mc>>1          /*arithmetic shift right*/
k=0
while (i>0)
begin
n=0
while (n<i)
begin
vₖ₊₁[n]=vₖ[n], vₖ[nₚ-n-1]
n=n+1
end
if nₚ is odd then
begin
vₖ₊₁[n]=vₖ[n]
n=n+1
end
nₚ=n
k=k+1
i=n>>1.          /*arithmetic shift right*/
end
```

Using this method with the example polynomial $Q(z) = 1 - z^{-2N}$ the intermediate coefficients are never larger than 2. In practice only a limited amount of intermediate coefficient growth occurs. This is not a very complex method (actually it only uses a different indexing) and delivers almost optimal results. For $P(z)$ the same procedure will be used only if m is even then $P(z)$ has a root at $z = -1$. If m is odd $P(z)$ does not have any additional roots, so no additional roots need to be added. The last step consists of using the equation

$$A_m(z) = \frac{P(z) + Q(z)}{2}$$

to compute the coefficients of $A(z)$ from $P(z)$ and $Q(z)$.